Synchronization of hyperchaotic harmonics in time-delay systems and its application to secure communication

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We present a predictor-feedback method for synchronizing chaotic systems in this paper. By using this method, two structurally equivalent or nonequivalent systems can be synchronized very effectively and quickly. Moreover, the feedback perturbation can be switched on even if trajectories of the two systems are far from each other. Therefore, this method is applicable to real-world experimental systems, especially to some fast experimental systems. The validity of this method is demonstrated by synchronizing hyperchaotic harmonics in a time-delay system. As an application, we introduce how messages can be encoded, transmitted, and decoded using this technique. We suggest taking use of the multistability of time-delay systems to improve the performance of the secure communication.

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I. INTRODUCTION

Synchronization of periodic signals is widely used in physical, engineering, and technical systems. Recently, chaotic synchronization [1] has become an issue of active research in light of its potential applications to secure communications [2–6]. An information signal containing some messages is transmitted using a chaotic signal as a broadband carrier. The scalar signal that is transmitted from the transmitter to the receiver is a function of the transmitter state variables and the information signal. If the receiver synchronizes with the transmitter, the information signal can be decoded exactly. Therefore, the key of secure communication is how to design a receiver system that can synchronize with the transmitter.

Different approaches of chaotic synchronization have been proposed. Pecora and Carroll [1] investigated the synchronization effect in a chaotic system that can be divided into a drive and a response subsystems, and they showed that a necessary condition for chaotic synchronization is that all the conditional Lyapunov exponents for the response system be negative. Another type of technique of synchronization was proposed by Pyragas [7] to synchronize two chaotic systems *A* and *B* by using an external feedback $x_n - y_n$, where x_n is the output of *A* and y_n is that of *B*. Because this approach is easily implemented in experiment, it is preferable in applications [8]. For some fast systems, x_{n+1} and y_{n+1}, however, may be far from x_n and y_n, thus this method still needs an amplitude limitation of feedback perturbation to solve the problem of large transient. More recently, Oliveira *et al.* [9] extended this technique using a predicted signal of *B* to synchronize two low-dimensional identical chaotic networks. The third type of method is the parameter perturbation method [10], which is based on the Ott-Grebogi-Yorke (OGY) [11] scheme for controlling unstable periodic orbits. However, this technique is only available when the chaotic trajectory points of two systems come close to each other within a small region. Therefore, how to develop a more practical synchronization method for a chaotic experimental system, especially a fast or/and high-dimensional system, is always considered to be a particularly interesting subject. In this paper, we will present a parameter perturbation technique based on the OGY method to synchronize a high-dimensional system.

Now let us return to the problem of secure communication. Some examples based on chaotic synchronization were presented in Refs. [2,3] that were based on simple low-dimensional chaotic systems with only one positive Lyapunov exponent. Recently, it has been shown that the hidden message can be decoded in such simple systems using some methods [3,12]. In order to improve security, synchronization of high-dimensional systems having multiple positive Lyapunov exponents (i.e., hyperchaotic systems) is preferable [4,5,13]. More recently, chaotic time-delay systems described by delay differential equations (DDE's) have been considered as good candidates for secure communication [6,14] because these systems can produce chaotic attractors with an arbitrarily large number of positive Lyapunov exponents [15], and some examples of synchronization of these systems have been offered [16,17]. A communication scheme based on the synchronization of chaotic laser diodes with delay feedback has recently been implemented experimentally [6].

Generally, a time-delay system related to an optical bistable or hybrid optical bistable device is described by

$$\tau' \dot{x}(t) = -x(t) + f(x(t-t_R), \mu), \tag{1}$$

where $x(t)$ is the dimensionless output of the system at time t, t_R is the time delay of the feedback loop, τ' is the response time of the nonlinear medium, and the parameter μ is pro-

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portional to the intensity of the incident light. In Eq. (1), $f(x,\mu)$ is a nonlinear function of x, characterizing the different system, e.g., $f(x,\mu)=\mu\pi[1-\zeta\cos(x-x_0)]$ for the Ikeda model [18], $f(x,\mu)=\pi[A-\mu\sin^2(x-x_0)]$ for the Vallée model [19], and $f(x,\mu)=\mu\sin^2(x-x_0)$ for the sine-square model [20]. Measuring the delay time in units of t_R, one can rewrite Eq. (1) as

$$\tau\dot{x}(t)=-x(t)+f(x(t-1),\mu), \tag{2}$$

where $\tau=\tau'/t_R$ characterizes the effect of the time delay when τ' is fixed. In this paper, we study Eq. (2) with a special feedback function $f(x,\mu)=1-\mu x^2$. Thus Eq. (2) can be rewritten as

$$\tau\dot{x}(t)=-x(t)+1-\mu x^2(t-1). \tag{3}$$

This special feedback function can be considered as the first nonlinear term of the Taylor expansion of the general nonlinear function $f(x,\mu)$ in the vicinity of a steady state. It should keep the general nonlinear properties of DDE's, as shown in Refs. [21,22]. Ikeda, Daido, and Akimoto [18] have reported odd-harmonic solutions in the Ikeda model in the long-time delayed case (i.e., the delay t_R is much greater than the response time τ' of the system), where each harmonic coexists with others and their oscillation periods are given by T_F/n, where T_F is the period of the fundamental solution and n is an odd integer. About the system (3), we have investigated in detail the stable regions of these odd harmonics in our previous work [22].

Equation (3) can be solved numerically and a fourth-order Adams' interpolation is suitable for that. Figure 1 illustrates the different chaotic harmonics at $\tau=0.02$ and $\mu=1.5$. These coexisting solutions are obtained from different initial functions at fixed parameters. In order to show the hyperchaotic characters of the system (3), we calculate the ten largest Lyapunov exponents at $\tau=0.02$, and μ is varied from 1.2 to 1.65, shown in Fig. 2. At $\mu=1.5$, the Kaplan-Yorke dimension of the system is 18.1, therefore it is hyperchaotic.

This paper is organized as follows. In Sec. II, the idea of the predictor-feedback method is presented. Then we apply this technique to a time-delay system and show some numerical results of the synchronization of hyperchaotic trajectories in Sec. III. Our numerical experiments show that this synchronization method is very efficient. In Sec. IV, we demonstrate how the message signals can be encoded, transmitted, and decoded by using this synchronization method in secure communication. Finally, a summary of this paper is concluded in Sec. V.

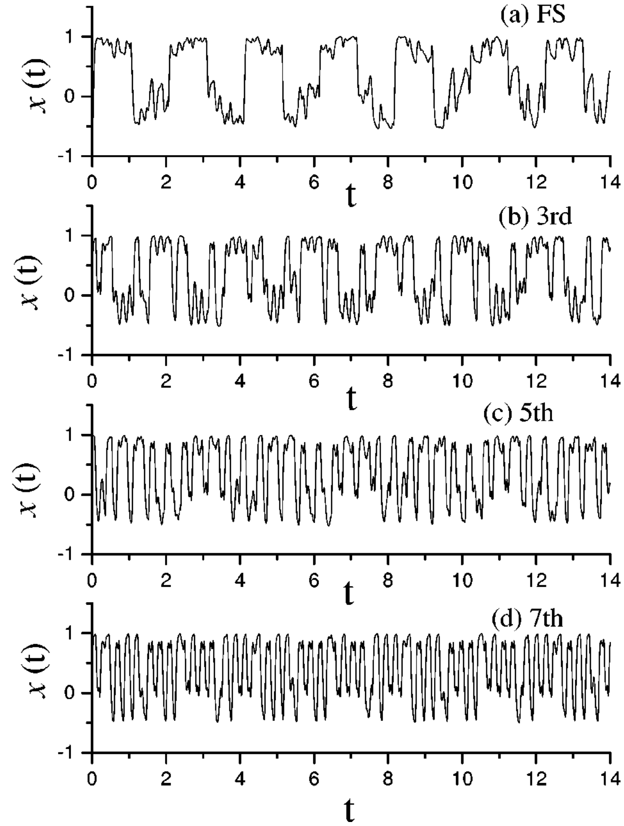

FIG. 1. The different solutions of Eq. (3) corresponding to the different harmonics for $\tau=0.02$ and $\mu=1.50$. (a) Fundamental solution (FS). (b) Third harmonic (3rd). (c) Fifth harmonic (5th). (d) Seventh harmonic (7th).

II. SYNCHRONIZATION METHOD

In 1992, Schwartz and Triandaf [23] proposed a predictor-corrector method to control and track unstable orbits based on the OGY method. Using a prediction step to determine parameter perturbation, they successfully ensured the next iterate of the system to accurately fall on the stable manifold of the desired object. Here we introduce a similar idea to develop a practical synchronization technique using parameter perturbation.

Most experimental systems are essentially ''black boxes'' for which one can only get a series of output signals $x_1,x_2,x_3,\ldots$, and measure some adjustable system parameters. Let us consider two almost identical chaotic experimental systems that we call A and B,

$$x_{n+1}=f(x_n,p_0), \quad A, \tag{4}$$

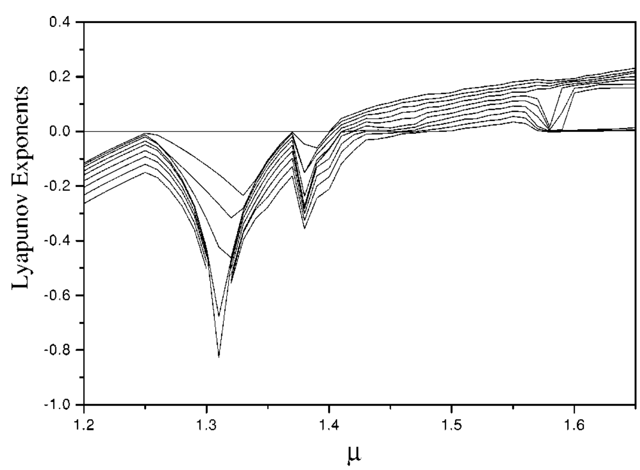

FIG. 2. Ten largest Lyapunov exponents of Eq. (3) versus bifurcation parameter μ at $\tau=0.02$. At $\mu=1.50$, the system is hyperchaotic.

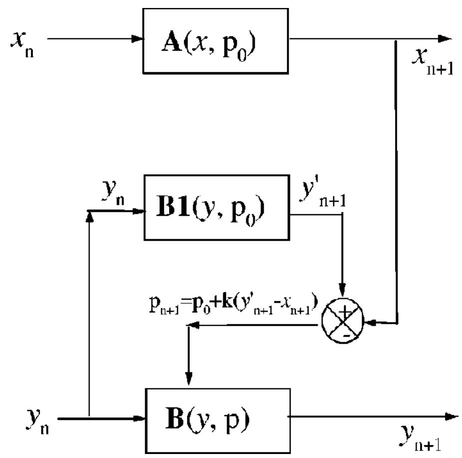

FIG. 3. Schematic illustrations of our strategy to synchronize two chaotic systems. A is the object system, B is the response system, and $B1$ is the predictor system. x and y are the corresponding measured outputs of A and B. y' is the prediction signal. For systems A and $B1$, $p=p_0$. For system B, p is the adjustable parameter.

$$y_{n+1}=f(y_n,p), \quad B, \tag{5}$$

where A is an object system and B is a response system. To synchronize B with A starting on different initial conditions, we imagine that p_0 for system A is a fixed parameter value and p for system B is an externally adjustable parameter. Our experimental design is schemed in Fig. 3. In this figure, x and y are the externally measured outputs of A and B, respectively. From this figure, one can see that there is another identical chaotic system $B1$ besides A and B. In our design, $B1$ is the system B in the case of no feedback perturbations (i.e., the value of parameter p in $B1$ is p_0). That is, the input of $B1$ is as the same as the input of B and the output is y'_{n+1}, which can be used to construct parameter perturbation to modify the response system B, so we call B used in this way a predictor system and denote it as $B1$. In a real-world experiment, we could use a computer to complete the task of $B1$. In the next section, we will find that because this predictor system is added in the design, one can avoid the limitation that the trajectory points of systems A and B must come close to each other when the feedback perturbation is activated [10].

As schemed in Fig. 3, the parameter perturbation for system B can be configured as

$$p_{n+1}=p_0+k(y'_{n+1}-x_{n+1}) \tag{6}$$

from the outputs of A and $B1$, where k is the feedback coefficient. Once k is suitably selected, the response system B will synchronize with the object system A. The suitable coefficient k can be obtained by scanning the coefficient interval, as done by Pyragas [7] in his control law. In addition, we have illustrated [24] another practical technique for experimental systems to select the coefficients in controlling chaos. In the case of chaotic synchronization, we can also use a similar technique to obtain k.

Here let us discuss the characteristic of feedback perturbation (6). A fundamental feature of perturbation signals (6) is that it does not change the solution of response system B,

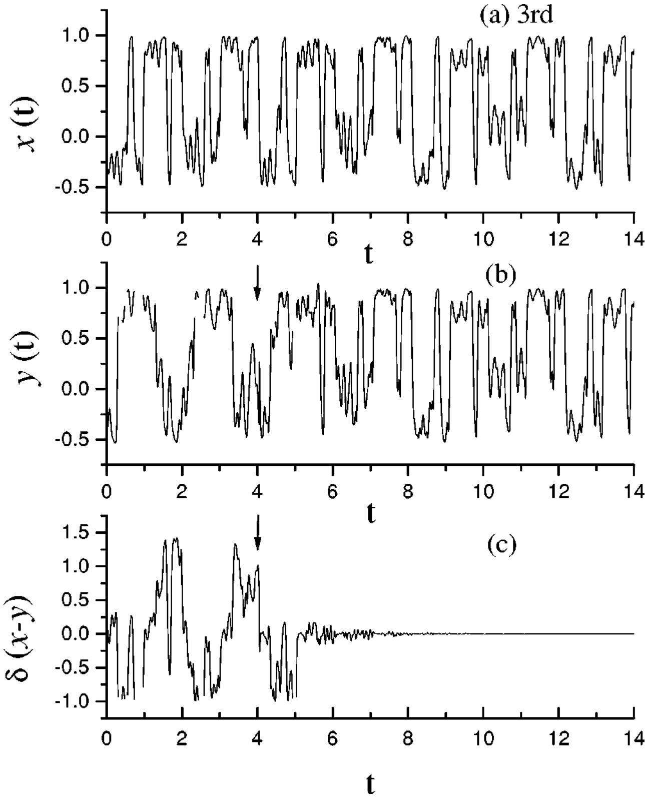

FIG. 4. Synchronizing two time-delay systems (3) at $\tau=0.02$ and $\mu=1.50$. Time traces of (a) the objector output $x(t)$, (b) the response output $y(t)$, and (c) the difference $\delta(x-y)$ before and after the feedback perturbation is activated. Initially, the object system is located in the third harmonic and the response system in the fundamental solution. The arrows mark the moment of switching onto the perturbation.

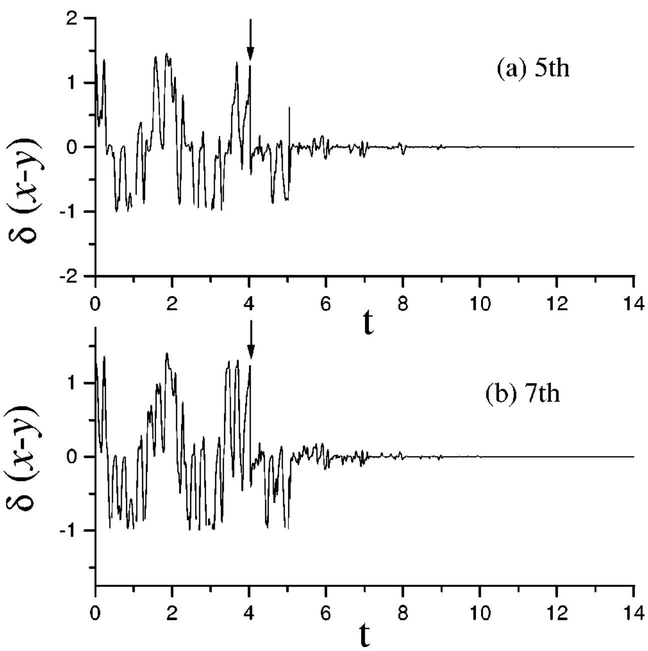

FIG. 5. Synchronizing results for different harmonics. Time traces of difference $\delta(x-y)$ before and after the feedback perturbation is activated. Initially, the object system is located in (a) the fifth harmonic, (b) the seventh harmonic, and the response system is located in the fundamental solution. Here $k=2.0$ and the arrows denote the same as in Fig. 4.

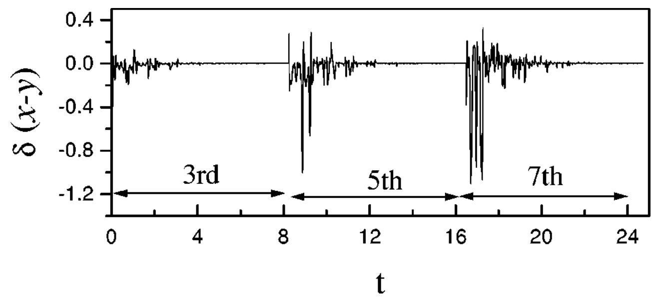

FIG. 6. Synchronizing result for the object system switching among different harmonics versus time t. Here $k=2.0$.

because when synchronization is achieved, A and B become completely uncoupled and at this time p is equal to p_0. Furthermore, this method has other advantages. (i) One can easily construct the feedback perturbation (6) only using the time series of the predictor and object systems and does not require Taken's delay-coordinate embedding technique or maximal correlation technique [25]. Thus it is well-suited for a ''black-box'' experimental system. (ii) This technique needs neither the trajectory points close to each other nor the amplitude limitation of perturbation when the feedback perturbation is activated. This gives us a convenient way for synchronizing real-world experiments (see the examples of the next section). (iii) This method can also be used to synchronize two structurally nonequivalent systems (i.e., systems generating chaotic attractors with high and different fractal dimensions) [16].

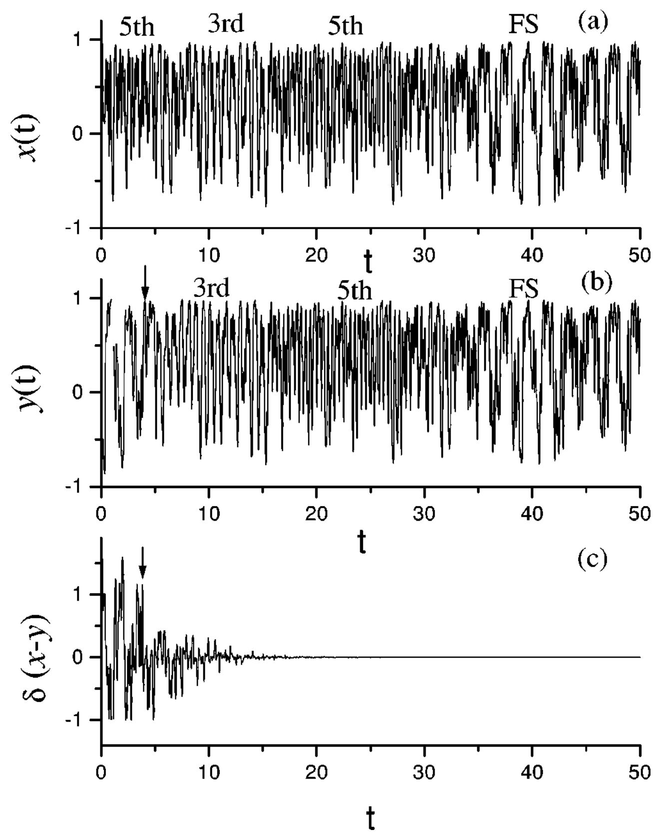

FIG. 7. Synchronizing result for the object system switching on the chaotic itinerancy solution at $\tau=0.04$ and $\mu=1.90$. Here $k=2.0$.

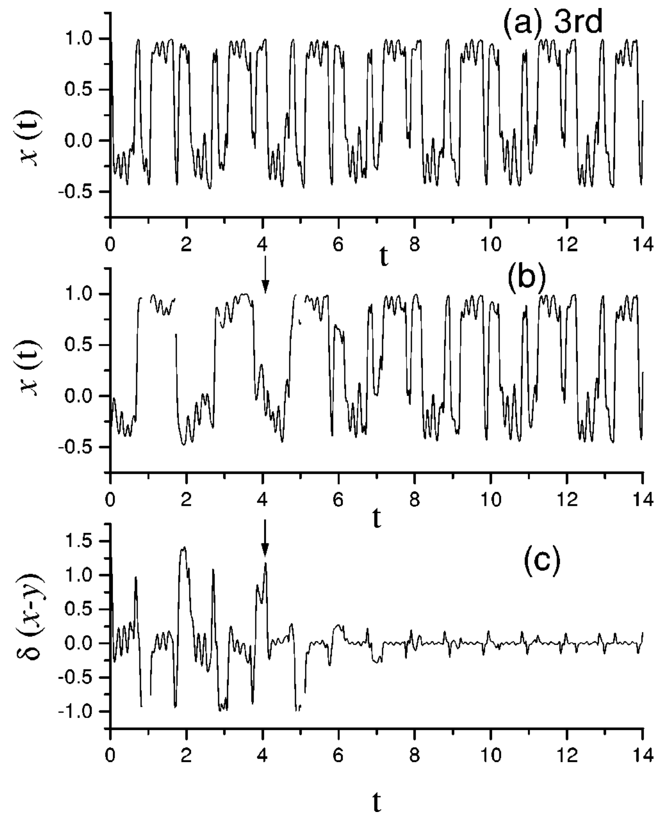

FIG. 8. Synchronizing results for two structurally nonequivalent systems (3). Time traces of (a) the objector output $x(t)$ with $D=18.1$ at $\tau=0.02$ and $\mu=1.50$, (b) the response output $y(t)$ with $D=14.3$ at $\tau=0.025$ and $\mu=1.50$, and (c) the difference $\delta(x-y)$ before and after the feedback perturbation is activated. The arrows mark the moment of switching onto the perturbation. Here $k=2.0$.

III. SYNCHRONIZATION OF CHAOTIC HARMONICS IN TIME-DELAY SYSTEMS

In order to introduce the above technique to a high dimensional time-delay system (3), we use

$$\mu(t+h)=\mu_0+k[y'(t+h)-x(t+h)] \tag{7}$$

to configure the parameter of response system B, where h is the integrated step (in this paper, $h=0.01$) and $y'(t+h)$ is the prediction signals of the predictor system.

As shown in Fig. 1, different solutions of Eq. (3) coexist at $\tau=0.02$ and $\mu=1.5$ (for the coexisting regions, interested readers are referred to Ref. [22]). Obviously, the fundamental solution and harmonics are located in the chaotic states. We configure the response system as described in the preceding section and use Eq. (7) as the parameter perturbation. Figure 4 shows time traces of the object output $x(t)$, response output $y(t)$, and the difference $\delta(x-y)$ before and after the parameter perturbation is turned on. An arbitrary value in the interval [0.6,5.5] can be used as the value of coefficient k to construct the feedback. In this figure, the initial state for the response system is selected at the fundamental solution and the object system is located in the third chaotic harmonic. At $t=4$, the feedback perturbation (7) with $k=2.0$ for the response system is activated, and the output of the response system quickly synchronizes with that

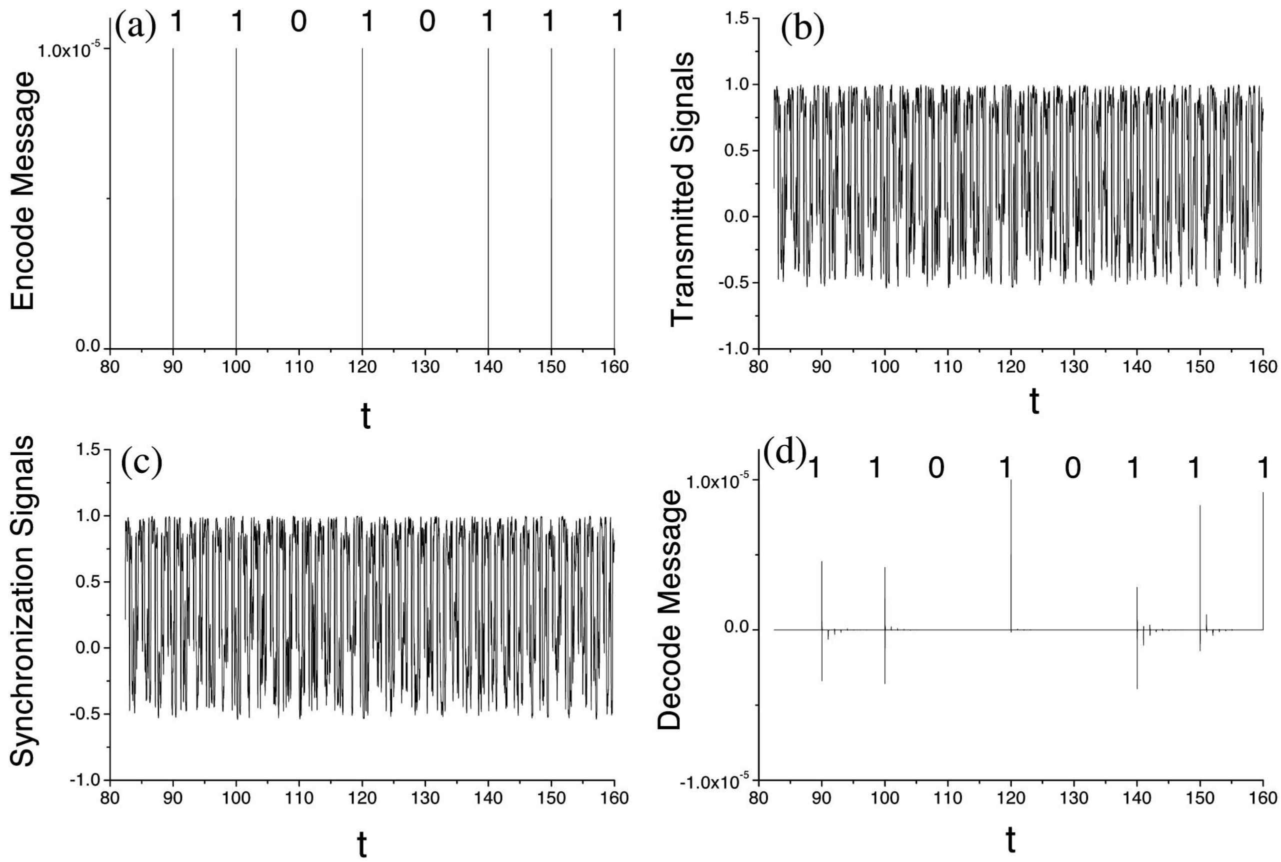

FIG. 9. (a) The encoded binary messages, (b) the transmitted chaotic time series (third harmonic), (c) the synchronized chaotic time series, and (d) the decoded messages versus time t. The binary sequence (10^{-5}) is produced at every $t=10$, masked in the third chaotic harmonic, and transmitted. (d) is the deviation of the transmitter signal and synchronization signal. Each spike corresponds to a binary signal ''1.''

of the object system. Unlike the method in Ref. [10], the initial difference of the two chaotic systems is quite large [in Fig. 4(c), $\delta(x-y)=1.0$] when the perturbation is activated. Figure 5 illustrates similar synchronization results, where the object system is located in the different harmonics [(a) fifth harmonic and (b) seventh harmonic] and the response system always starts at the fundamental solution. In fact, the response can also be chosen as other initial states (e.g., the third harmonic or the fifth harmonic, etc.). Once the perturbation with a suitable feedback coefficient is activated, the response quickly switches onto the corresponding state of the object system automatically and the difference between them quickly tends towards zero.

To show that, in Fig. 6, the response and object systems are located in the fundamental and the third-harmonic states, respectively, and then the perturbation is activated. One can see that the response system quickly synchronizes with the object system to the third-harmonic state. At $t=8$, we shift the object system onto the fifth-harmonic state, the response system also synchronizes with it to the fifth harmonic automatically, and at $t=16$ the object is switched onto the seventh harmonic; the response follows, as shown in Fig. 6. Here what we should emphasize is that the different harmonic states in Fig. 6 coexist with other harmonics in the same parameters (i.e., $\tau=0.02,\mu=1.5$). Due to the multistability and hyperchaos of time-delay systems, we expect that a more secret way for communication may be obtained by synchronizing two identical delay systems. On the one side, a hyperchaotic transmitter ensures a highly efficient level of security. On the other side, switching of different coexisting harmonics can also provide us with a multichannel way to transmit messages.

Another idea that can be used in secure communication is the chaotic itinerancy solution of time-delay systems. Chaotic itinerancy means that a dynamical system switches among different unstable local chaotic orbits on a time scale, compared to the dynamics on each attractor ruin. At $\tau=0.04$ and $\mu=1.9$, we can observe a chaotic itinerancy solution among the fundamental solution, third harmonic, and fifth harmonic (see Ref. [22]). In Fig. 7, the object system is initially located in the chaotic itinerancy solution and the response system starts from the fundamental solution. Once the feedback perturbation is activated, the response system synchronizes with the object quickly and is also found in the itinerancy solution, as shown in Fig. 7.

At the end of this section, we should point out this predictor-feedback method can also be used to synchronize two structurally nonequivalent systems [16]. In Fig. 8, we initially configure the object system to the third harmonic with $D=18.1$ (here D is the Kaplan-Yorke dimension) at $\tau=0.02$ and $\mu=1.50$, and the response system to a fundamental solution with $D=14.3$ at $\tau=0.025$ and $\mu=1.50$. Once the predictor-feedback perturbation is activated at $t=4$, the response system quickly follows the object system to the third harmonic as shown in Fig. 8(b). Figure 8(c) illustrates the difference of the two nonequivalent systems.

IV. APPLICATIONS TO SECURE COMMUNICATIONS

Pecora and Carroll [1] have mentioned that one of the possible applications of the chaotic synchronization is secure communication. In experiment, we add a signal generator and an information-encoding system as accessory systems to the object system *A*, and all of them are organized as the transmitter system. The receiver system is made up of a decoding system, the predictor $B1$, and the response *B*. The information signal containing messages (i.e., a series of binary signals) is encoded in the output of *A*, and all of them are transmitted. In general, the information signal is very small compared to the amplitude of the chaotic output. When *B* synchronizes with *A*, we can then decode the binary messages by detecting deviation between outputs of *B* and the transmitter.

In Fig. 9, the information signal to be transmitted is a binary sequence (i.e., 1,1,0,1,0,1,1,1, . . .) and each binary element is transmitted at every $t=10$. In this example, the binary sequence is encoded in the third chaotic harmonic and the amplitude of the sequence is 10^{-5}. Figure 9(b) is the transmitter signal that contains the message signals. Clearly the small binary sequence is masked by the chaotic harmonic. Figure 9(c) shows the synchronization signal of the response system *B* and Fig. 9(d) shows the deviation between the transmitter signal and synchronization signal versus time. From Fig. 9(d), one can see that each spike corresponds to a binary signal ''1.'' Thus one can accurately decode the message signals by detecting the spikes.

We should emphasize that the synchronization results of the preceding section regarding switching among different harmonics and chaotic itinerancy can be used in secure communications. From Figs. 6 and 7, we know that the response system can synchronize with the object system so quickly. Once synchronization is obtained, the above idea can be easily applied to improve the security of communication.

V. CONCLUSIONS

In conclusion, we presented a practical perturbation technique (i.e., predictor-feedback method) for synchronizing fast or/and high-dimensional systems. First, this method is designed for experimental situations in which we have no analytical knowledge of the system dynamics (i.e., ''black-box'' system). We only used a series of experimental outputs to construct the parameter-perturbation to synchronize two chaotic systems easily. Second, with a predictor system, the response system can quickly synchronize with the object system automatically whenever the parameter perturbation is activated. That is, even though the trajectory point of response system is far from the trajectory of object system, the predictor feedback method is still highly effective because it does not require limitation of perturbation amplitude. We showed the validity of this technique by synchronization of high frequency harmonics in a time-delay system. Third, this method can also be used to synchronize two structurally nonequivalent systems.

In order to apply this technique to secure communication, we designed a strategy to encode, transmit, and decode the information signal by using this method. Considering that the odd harmonics of time-delay system are not only hyperchaotic but also coexist with each other, we believe that chaotic time-delay system is a good candidate for secure communication. On the one hand, the hyperchaotic attractors with a large number of positive Lyapunov exponents improve the secure ability for communication. On the other hand, the coexisting behavior of odd harmonics can provide us a new way for chaos communication with low detectability. In addition, the chaotic itinerancy solution in time-delay systems also give us another way for secure communication.

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